

Endterm Test (sample) – paper part. 20 points (≥ 8 required). Nothing but pen is allowed.

1. a) What is a zero divisor? What are the zero divisors of $\mathbb{Z}_3$ and $\mathbb{Z}_4$? (2 p)

b) Is $\mathbb{N}$ a ring? Is $\mathbb{R}^{2 \times 2}$ a ring? If not, which property is not satisfied? (2 p)

c) Calculate $81^{62} \bmod 25$ using Euler's totient theorem. (4 p)

d) Calculate $(\bar{1} + \bar{2}\theta)^3$ in $\mathbb{F}_{25} = \mathbb{Z}_5(\theta)$, where θ 's minimal polynomial is $x^2 - \bar{3}$. (4 p)

2. Calculate the quadratic polynomial p (in canonical form) for which: (8 p)
 $p(1) = -2$, $p(2) = -2$, $p(4) = 4$. (Check the result.)