Divisors, prime numbers

Exercises

Application of discrete models

Mathematical questions

1. Write down the division theorem. Demonstrate it for 21 and 5.

2. Define divisors. Demonstrate the definition for any divisor of 12.

3. List the divisors of 16. Which of them are proper divisors and/or nontrivial divisors?

4. Define what associate means. What are the associates of 10?

5. Define prime numbers. List all prime numbers up to 15.

6. How are the numbers with more than two positive divisors called? Give an example.

7. What does the fundamental theorem of arithmetic state?

8. Give the prime factorization (canonical representation) of the following numbers:
a) 30, b) 36, c) 37, d) 64, e) 150, f) 200, g) 3000.

9. What is $\tau(n)$ and $\sigma(n)$? What is $\tau(12)$ and $\sigma(12)$?

10. Calculate the following values – in at least two ways:
a) $\tau(7)$, b) $\tau(8)$, c) $\tau(9)$, d) $\tau(10)$, e) $\tau(12)$, f) $\tau(24)$, g) $\tau(32)$, h) $\tau(300)$.

11. What is a perfect number? Is 28 a perfect number?

Programming exercises

12. Solve the following exercises using the built-in prime-related functions of Sage – in at least three different ways for each one.

a) Print all prime numbers between 1 and 100.

b) Print the first 100 prime numbers.

c) How many prime numbers are there below 1000?

13. Write a function that decides whether a given $n \in \mathbb{N}$ is prime (without using the prime number functions of Sage). Check the solution using `is_prime()` from 0 to 100, and check it also for the following prime numbers: 1237, 1212121, 1234567891.

14. Write a function that returns the list of positive divisors of a number (without using the mathematical functions of Sage). Check the solution using `divisors()` from 1 to 100, and check it also for the following numbers: 1000, 1111111, 2^{30}.

15. Check for all numbers between 1 and 100 that divisibility is transitive.

16. Create the list of all twin prime pairs below 600: $[(3,5),\ (5,7),\ (11,13),\ \ldots]$.

17. Write a function that creates all prime numbers up to a given n, using the sieve of Eratosthenes. (Don't use the built-in prime-related functions.) The function returns the list of prime numbers (which is created from the `bool` list of the algorithm at the end). Call it for $n = 100$, and print the resulting list. Call it also for $n = 100000$, and check whether the last prime number is correct (using `previous_prime()`).

18. a) Write a function that calculates $\tau(n)$ using the definition. The `number_of_divisors()` function can be used for testing.

b) Write a function that calculates $\tau(n)$ using n's prime factorization.

c) Print those n's between 1 and 100 for which $\tau(n)$ is odd. What is the rule? Why?

19. Find all perfect numbers up to 10000.