SageMath introduction

Exercises

Application of discrete models

1. Calculate the exact value (which is an integer):

$$\sqrt{\frac{\left(\tan\left(-\frac{7}{12}\pi\right)-2\right)^2}{\frac{123456}{\left(\frac{1}{2}\right)^{-13}+9!-8\cdot 88}-2^{\binom{42}{5}-(2^8+1)\cdot 3310}}}$$

2. Create the following lists:

a) powers of two from 1 to 2^{20};

b) factorials from 0! to 20!: $[1, 1, 2, 6, 24, \ldots]$;

c) powers of x up to 100: $[1, x, x^2, x^3, \ldots, x^{100}]$;

d) the 10^{th} row of Pascal's triangle;

e) 10 lists, where the n^{th} list is the n^{th} row of Pascal's triangle: $[[1], [1, 1], [1, 2, 1], \ldots]$;

f) the sines of the degrees $0°, 30°, \ldots, 360°$: $[0,\ 1/2,\ \sqrt{3}/2,\ \ldots,\ 0]$;

g) $\sin(2x)$ and its first 20 derivatives (what is the rule?).

3. Create the following matrices:

a) times table (i.e. $a_{ij} = i \cdot j$) from 1 to 10;

b) the even numbers from 2 to 200 sequentially in a 10×10 matrix;

c) 10×10 identity matrix (i.e. ones in the main diagonal, and zeros everywhere else);

d) first 10 rows of Pascal's triangle (extended by zeros to fill the square);

e) Vandermonde matrix for a given input vector, e.g. for (a_1, a_2, a_3, a_4), the following matrix:

$$\begin{pmatrix} 1 & a_1 & a_1^2 & a_1^3 \\ 1 & a_2 & a_2^2 & a_2^3 \\ 1 & a_3 & a_3^2 & a_3^3 \\ 1 & a_4 & a_4^2 & a_4^3 \end{pmatrix}$$

(write it as a function, whose parameter is the input vector of any size).

4. a) Write a function that calcalates the sum of the n^{th} row of Pascal's triangle for a given n.

b) Collect these sums into a list from $n = 0$ to 10.

c) What is the rule? Check your guess from $n = 0$ to 100.

5. Add up the numbers between 1 and 12345 which are divisible by 3 – in at least three different ways. Put each solution into a separate function. They don't receive any parameter, and return (not print!) the sum.

6. Calculate the following values from $n = 0$ to 10, and put them into a list for each part (i.e. each list will contain 11 integers):

a) the Fibonacci numbers using the Sage function `fibonacci(n)`;

b) the elements of the following recursive sequence: $F_0 := 0,\ F_1 := 1,\ F_n := F_{n-1} + F_{n-2}$;

c) for $F := \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$, the top right element of each F^n;

d) $\sum_{k=0}^{\lfloor \frac{n-1}{2} \rfloor} \binom{n-k-1}{k}$, where $\lfloor \cdot \rfloor$ denotes the floor function (notice how the summation is on specific "oblique" diagonals of Pascal's triangle);

e) the exact values (not approximations!) of $\frac{\varphi^n - (1-\varphi)^n}{\sqrt{5}}$, where $\varphi := \frac{\sqrt{5}+1}{2}$ is the golden ratio (don't leave square roots in the result – e.g. use `(...).simplify_full()` to get rid of them).