Discrete Mathematics I, Endterm test A
Autumn 2024

NAME: NEPTUN code:
Class Teacher:

Duration: 90 minutes.

Total score: The maximal total score achievable in the Endterm test is **40 points**.

Success criterium: In order to successfully pass this test, you need to achieve at least **16 points**.

Equipment: Only blank papers and pen allowed, no calculators.

Instructions:
Each task must be solved on paper using pen. Please

- write your name, Neptun code and the name of your practice class teacher on the top of this test paper;
- write your name and Neptun code on the top of each paper you use;
- after finishing the test, place all the papers with your solutions behind this test paper and fold them into half parallel to the longer side, in the middle (so that your papers stay together);
- justify your answer to each question, except for Question 1. Brief justifications are sufficient. Just an answer to the questions without any justification is worth very few marks only (except for Question 1), so please do not forget to justify your answers;
- a 'yes' or no' answer alone without any justification is worth 0 marks;
- you do not need to work out the answers as single numbers: you can leave binomial coefficients or factorials of large numbers in your answers, you do not need to calculate these (hence a calculator is unnecessary);
- note that this test paper will be shared on Canvas, hence you will be able to obtain it.

Thank you.

Questions

1. In this question proofs are not required, writing down the answers only is sufficient.

   (a) In a running race there are 24 participants. How many different outcomes are possible for the first 5 places?

   (b) In how many different ways can 5 identical gifts be distributed among 12 people, if each person can get at most one gift?

   (c) How many different strings can be formed using exacly the following letters: $A, A, A, B, C, C, D, D, D$?

   (d) In how many different ways can 12 different gifts be distributed among 7 people, if anyone can get any number of gifts?

   (e) In how many different ways can 20 people sit down around a round table? (Those seatings which can be transformed into each other by rotation are considered identical.)

   (f) In how many different ways can 8 apples be distributed among 11 children? The apples are identical and any child can get any number of apples.

   (g) At least how many people do we need to have in a group in order to be certain that there are 3 among them who were born in the same month of the year? **(7 marks)**

2. The administration office at the Faculty of Informatics is planning to purchase 100 mouse pads in total. There are four different types of mouse pads available to choose from.

   (a) In how many different ways can they choose the 100 mouse pads? (An example for a possible choice: they can buy for example 12, 26, 51 and 11 mouse pads of type 1, 2, 3 and 4, respecively.)

   (b) In how many different ways can they choose the 100 mouse pads if they would like to order at least 5 mouse pads of each type? (We do not distinguish between mouse pads of the same type.) **(3 marks)**

3. Boris has 10 books and he would like to put them all on one book shelf in a row. Three of these books have the following titles: 'War and peace', 'Pride and prejudice' and Crime and punishment'. In how many different ways can Boris arrange the ten books into order so that

   (a) 'War and peace' and 'Pride and prejudice' are *not* next to each other?

   (b) 'War and peace', 'Pride and prejudice' and Crime and punishment' are next to each other in some order?

   (c) 'War and peace' is the first one in the row of books and Crime and punishment' is *not* the last one in the row? **(7 marks)**

4. (a) How many different 7-digit numbers exist consisting exactly of the following digits: 0, 1, 2, 3, 4, 4, 5?

   (b) How many of the above 7-digit numbers are divisible by 5?

   (c) How many different 7-digit numbers exist consisting exactly of the following digits: 0, 1, 2, 3, 4, 4, 5 such that the last digit is not 4? **(9 marks)**

5. (a) In the expansion of $(4x^5 - \frac{2}{x^3})^{10}$ find the coefficients of the terms x^{50} and x^{27}.

   (b) A secondary school is attended by 150 students. In the school there are three types of extracurricular activities offered: chess, volley ball and badminton. Among all the students there are 45 doing chess, 54 playing volley ball and 72 playing badminton. There are 17 students playing both chess and volley ball, 26 doing both chess and badminton and 21 playing both volley ball and badminton. There are 37 students who are not playing any of these games. How many students are taking part in all of the three activities? **(7 marks)**

6. For each of the degree sequences below decide if there exists

   (1) a graph,
   (2) a *simple* graph

   with the given degree sequence. In each case if such a graph exists, draw an example for it, otherwise prove that it does not exist.

   (a) 5,4,3,3,3,2,2

   (b) 6,4,4,4,4,2,0

   (c) 6,5,4,4,3,2,1 **(7 marks)**