Discrete Mathematics I, Sample midterm test
Spring 2025

NAME: NEPTUN code:

Group number/Class teacher:

Duration: 90 minutes.

Total score: The maximal total score achievable in the midterm test is **40 points**.

Success criterium: In order to successfully past this test, you need to achieve at least **16 points**.

Equipment: Only blank papers and pen allowed, no calculators.

Instructions:
Each task must be solved on paper using pen. Please

- write your name, Neptun code and the name of your practice class teacher on the top of this exam paper;
- write your name and Neptun code on the top of each paper you submit;
- note that in most questions **justification is required**. Just an answer to these questions without any proof is worth very few marks only. **Please do not forget to justify your answers**. (Applying and showing the steps of a method learnt in the class – where relevant – is regarded as sufficient justification.)
- note that a 'yes' or no' answer on its own without any justification is worth 0 marks only;
- after finishing the test, place all the papers with your solutions behind this exam paper and in order for the papers to stay together please fold them into half (parallel to the longer side);
- note that the actual test paper will be shared on Canvas, hence you will be able to obtain it.

Thank you and all the best for the test!

Questions

1. (a) On the set of all people consider the following predicates: $L(x)$: 'x is a lawyer'; $C(x)$: 'x is crafty' and $F(x,y)$: 'x and y are friends'. (We assume friendships to be mutual, i.e. $F(x,y)$ holds if and only if $F(y,x)$ is true.)
   i. Express the following statement using a formula: Every lawyer has a crafty friend, but it is not true that every friend of every lawyer is crafty.
   ii. Write down the negation of the above statement using a formula. If you negate the formula you gave in part 1(a)i, please perform at least one step of simplification on the negated formula.

   (b) For each of the equalities below decide if it is true for *every* set A, B and C. Prove your answers.
   i. $A \setminus (B \cup C) = (A \setminus B) \cup (A \setminus C)$.
   ii. $A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C)$. **(7 marks)**

2. Consider the binary relations $R = \{(1,4),(1,5),(2,7),(3,2),(6,1),(6,6),(7,4),(7,5),(7,7)\}$ and $S = \{(1,4),(2,1),(3,7),(4,1),(4,6),(5,3),(5,4),(5,5),(6,6)\}$. Find each of the following:

   (a) $dmn(S) \triangle rng(R)$,

   (b) $R(\{0,1,4,5,6,8\})$,

   (c) $R^{-1}|_{\{2,4,5,7\}}$ and

   (d) $S \circ R$. **(7 marks)**

3. (a) Let $\rho = \{(x,y) \in \mathbb{Z} \times \mathbb{Z} \mid 2x+1 = 3y-5\}$. Find $\rho(\{-1,0,1,2,3,4\})$ and $\rho^{-1}(\{-1,0,1,2,3,4\})$.

   (b) Given $R = \{(x,y) \in \mathbb{R} \times \mathbb{R} \mid 6x-1 = \frac{4y-3}{2}\}$ and $S = \{(x,y) \in \mathbb{R} \times \mathbb{R} \mid y^2+2 = 7x\}$ find the composition $S \circ R$. **(7 marks)**

4. (a) Consider the binary relation $R = \{(1,1),(1,2),(1,3),(2,1),(2,2),(2,4),(3,1),(3,3),(4,2)\}$ on set $X = \{1,2,3,4\}$. Decide, whether R is reflexive, symmetric, transitive and/or anti-symmetric. Justify each of your answers.

   (b) On set $A = \{1,2,3,4,5\}$ construct a relation R which satisfies all of the following properties: it is not transitive, not symmetric, not anti-symmetric and $R(\{4\}) = \{1,5\}$. **(6 marks)**

5. (a) For each of the following relations decide if it is an equivalence relation. Prove your answers.

      i. $R_1 = \{(x,y) \in \mathbb{R} \times \mathbb{R} \mid xy \geq 0\}$ on $\mathbb{R}$

      ii. $R_2 = \{(x,y) \in \mathbb{Z} \times \mathbb{Z} \mid 4 \mid x-y\}$ on $\mathbb{Z}$

   (b) For each of those relations above which are equivalence relations, find the partition determined by the equivalence relation. **(6 marks)**

6. (a) For each of the following relations decide if it is a function. Please, prove your answers.

      i. $f_1 = \{(x,y) \in \mathbb{R} \times \mathbb{R} \mid 2x-3 = y^2\} \subseteq \mathbb{R} \times \mathbb{R}$

      ii. $f_2 = \{(x,y) \in \mathbb{R} \times \mathbb{R} \mid 2\cos y = x\} \subseteq \mathbb{R} \times \mathbb{R}$

      iii. $f_3 = \{(x,y) \in \mathbb{R} \times \mathbb{R} \mid y - x^2 = 5\} \subseteq \mathbb{R} \times \mathbb{R}$

   (b) For each of the above relations which are functions, decide if it is injective, surjective and/or bijective. Prove your answers. **(7 marks)**