# Polynomials

(exercises)

Application of discrete models

1. Create the following polynomials for arbitrary $n \in \mathbb{N}^+$. Print the results for $n = 5$. (The type of the result is polynomial, not general symbolic expression.)
   a) $nx^n + (n-1)x^{n-1} + (n-2)x^{n-2} + \ldots + 2x^2 + x$;
   b) $x^{2n} + x^{2n-2} + \ldots + x^4 + x^2 + 1$;
   c) whose roots are $1, 2, 3, \ldots, n$ (all simple);
   d) whose only root is 2, of multiplicity $n$;
   e) which has the simple root 1, the double root 2, …, the root $n$ of multiplicity $n$.

2. Write a function that calculates the value of a polynomial $p$ at $c$ using Horner's method. (Use only the coefficients of the polynomial.)

3. Modify the previous function so that is calculates both the quotient and the remainder of $p$ divided by $x-c$ (in the same format as `p.quo_rem(x-c)`).

4. Find an integer $c$ for which $-1$ is a double root of $x^5 - cx^2 - cx + 1$.

5. Find an integer $c$ for which $x^2 + cx + c^2$ divides $x^5 + 3x^3 - x^2 + 14x + 4$.

6. a) Implement the Euclidean algorithm for polynomials in $\mathbb{Q}[x]$.
   b) Implement the extended Euclidean algorithm for polynomials in $\mathbb{Q}[x]$.
   c) Try both algorithms with $2x^3 + x^2 - 2x - 1$ and $x^2 - 2x + 1$, and check the results using Sage functions.

7. Determine the common roots of $2x^3 + x^2 - 2x - 1$ and $x^2 - 2x + 1$ without calculating all the roots of these polynomials.

8. Write a function that calculates the formal derivative of a polynomial. Use only its coefficients, not any polynomial operation.

9. a) Create a degree 6 polynomial $p$ which has the simple root 5, the double root 6 and the triple root 7. (The type of the result is polynomial.) Check $p$ using `roots()`.
   b) Calculate the roots of $\gcd(p, p')$. What is the relation to the roots of $p$?
   c) Calculate the roots of $p \,/\, \gcd(p, p')$. What is the relation to the roots of $p$?

10. Write a function that gets a polynomial $p$ and a value $c$, and it calculates both $p(c)$ and $p'(c)$ using Horner's method, without calculating the polynomial $p'$.