Greatest common divisor

(exercises)

Application of discrete models

1. For given $a, b \in \mathbb{N}^+$, calculate
   a) $\gcd(a, b)$, b) $\operatorname{lcm}(a, b)$
   from the prime factorization of a and b.

2. Implement the Euclidean algorithm: write a function that calculates $\gcd(a, b)$ for $a, b \in \mathbb{N}^+$,
   a) using subtraction; b) using remainders.

3. Calculate the following by hand, using the Euclidean algorithm:
   a) $\gcd(39, 15)$; b) $\gcd(32, 20)$; c) $\gcd(39, 30)$; d) $\gcd(157, 15)$; e) $\gcd(200, 120)$;
   f) $\gcd(125, 90)$; g) $\gcd(36, 10)$; h) $\gcd(102, 72)$; i) $\gcd(55, 34)$; j) $\gcd(700, 300)$.

4. Write a function that calculates $\operatorname{lcm}(a, b)$ from $\gcd(a, b)$.

5. Implement the extended Euclidean algorithm: write a function that, for given $a, b \in \mathbb{N}^+$, calculates $\gcd(a, b)$ and the Bézout coefficients for a and b.

6. On an island, there are 7-headed and 11-headed dragons. The total number of dragon heads is 160. How many dragons of each type are on the island? Solve the problem generally: write a function that receives $a, b, c \in \mathbb{N}^+$, where the dragons have a or b heads, and the total number of heads on the island is c; the function returns all possibilities in a list, where each element is a pair (the number of a-headed and b-headed dragons, respectively). For example, it returns $[(4, 12), (15, 5)]$ for the above question. (Don't use brute force search.)