Divisors, prime numbers

(exercises)

Application of discrete models

1. Write a function that decides for given $a, b \in \mathbb{N}^+$ numbers whether a is a divisor of b. Solve it in at least three ways. Use the built-in function `a.divides(b)` only for testing: check the solution for all integers `a` and `b` between 1 and 100.

2. Write a function that returns the list of positive divisors of a number. Don't use the built-in function `divisors()` other than for testing.

3. Check for all numbers between 1 and 100 that the divisibility is transitive.

4. Write a function that decides whether a given $n \in \mathbb{N}$ is prime. Don't use the built-in number theory functions of Sage. Check the solution using the built-in function `is_prime()` from 0 to 1000.

5. Print all prime numbers between 1 and 100 – in at least three different ways.

6. Print the first 100 prime numbers – in at least three different ways.

7. How many prime numbers are there up to 1 000 000? Solve it in at least three different ways.

8. Create a list that contains all twin prime pairs up to 1000: $[(3, 5),\ (5, 7),\ (11, 13),\ \ldots]$.

9. Implement the sieve of Eratosthenes: write a function that returns all prime numbers in a list up to a given n. (Don't use the built-in prime-related functions.)

10. Write a function that calculates $\tau(n)$
    a) using simple counting; b) from the prime factorization of n.

11. Write a function that calculates $\sigma(n)$
    a) using simple summation; b) from the prime factorization of n.

12. Find all perfect numbers up to 10000.