

Discrete Mathematics I, Sample Endterm test

Spring 2024

NAME:
Class Teacher:

NEPTUN code:

Duration: 90 minutes.

Total score: The maximal total score achievable in the Endterm test is **40 points**.

Success criterium: In order to successfully pass this test, you need to achieve at least **16 points**.

Equipment: Only blank papers and pen allowed, no calculators.

Instructions:

Each task must be solved on paper using pen. Please

- write your name, Neptun code and the name of your practice class teacher on the top of this test paper;
- write your name and Neptun code on the top of each paper you use;
- after finishing the test, place all the papers with your solutions behind this test paper and fold them into half parallel to the longer side, in the middle (so that your papers stay together);
- justify your answer to each question, except for Question 1. Brief justifications are sufficient. Just an answer to the questions without any justification is worth very few marks only (except for Question 1), so please do not forget to justify your answers;
- a 'yes' or no' answer alone without any justification is worth 0 marks;
- you do not need to work out the answers as single numbers: you can leave binomial coefficients or factorials of large numbers in your answers, you do not need to calculate these (hence a calculator is unnecessary);
- note that this test paper will be shared on Canvas, hence you will be able to obtain it.

In Question 6 you have a choice between two questions: You do not need to solve both of them, please choose just **one** of them.

Thank you.

Questions

1. In this question proofs are not required, writing down the answers only is sufficient.

A class of 20 pupils are going on a trip.

- (a) They organize a running race. How many different outcomes are possible for the last 4 places, if we assume that there are no equal finishes and all the 20 participants reaches the finish?
- (b) In how many ways can they be allocated to 4 different rooms if each room can take 5 people?
- (c) In how many different ways can they sit in a circle around the bonfire in the evening? (Two seatings are regarded as identical if they can be transformed into each other by rotation.)
- (d) In how many different ways can they select on a given day those 5 pupils who are going to do the shopping on that day?
- (e) In how many different ways can they elect a committee of 6 members which contains a leader, a deputy leader and 4 ordinary members?
- (f) In how many different ways can 30 apples be distributed among them if the apples are all identical and any pupil can get any number of apples? (We do not cut the apples into pieces, so everyone gets a whole number of apples.)
- (g) In how many different orders can they sit in a row of 20 seats in the cinema, if Andrew and Belinda would not like to sit next to each other? **(7 marks)**

2. On a production line in a factory 200 locks were manufactured during a shift. 7% of these locks were faulty. In how many different ways can we choose 30 locks from among the 200 locks so that
- there are exactly 7 faulty locks among them?
 - there are at least 3 faulty locks among them?
- (5 marks)**
3. (a) There are 4 types of candy in a box: apple, orange, vanilla and menthol flavoured ones (at least 25 pieces of each type). Candies of the same flavour are all identical. In how many different ways can Brian select 25 candies, if he would like to take at least three of each kind of candy? (It does not matter what order he picks the candies.)
- (b) How many different 6-digit numbers can be formed using exactly the digits 0, 1, 2, 5, 6, 6 which are divisible by 4? (Please note that each digit needs to be used exactly as many times as it appears in the list.)
- (6 marks)**
4. Allen has 10 different books at home and he would like to arrange them on his book shelves. In how many different ways can he do this
- if he would like to arrange all the books on one shelf in a row so that the book: 'Robinson Cruso' cannot be the first nor the last book in the row?
 - if he would like to arrange the books on two shelves putting 6 books on the top shelf and 4 books on the bottom shelf (in rows) so that the book 'War and peace' has to be on the top shelf and 'SPSS' has to be on the bottom one?
 - if he would like to arrange the books on three shelves putting 3 books on the top shelf, 3 books on the middle shelf and 4 books on the bottom shelf (in rows) so that 'C Programming' and 'SPSS' are both on the top shelf and the book 'Harry Potter and the Philosopher's Stone' cannot be on the bottom shelf?
- (8 marks)**
5. (a) In the expansion of $(3x^2 + \frac{2}{x})^8$ find the coefficients of the terms x^{18} and x^7 .
- (b) On a Winter day there are 120 people in a park: 55 of them are wearing hats, 44 of them are wearing scarves and 32 of them are wearing gloves. There are 14 people wearing both hats and scarves, 12 wearing both hats and gloves and 7 are wearing both scarves and gloves. There are 17 people who are not wearing any of these items. How many people in the park are wearing all these three pieces of clothing?
- (7 marks)**

There are two versions of Question 6, which you can choose from. Please, choose only **one** of the two versions. Even in case you solve both versions, you will obtain marks only for one of them.

Version 1:

6. Represent the following sets in the Gaussian plane:

- $\{z \in \mathbb{C} \mid \operatorname{Re} z + 1 > \operatorname{Im} z\}$
 - $\{z \in \mathbb{C} \mid |z - 1 - i| \geq 2 \wedge \operatorname{Im} z < 2\}$
- (7 marks)**

Version 2:

6. Can a graph (i.e. undirected graph) have the following degree sequence? And a *simple* graph? At each question, if the answer is yes, please, justify this by drawing such a graph. If the answer is no, explain why it is not possible. (Please note that a 'yes' or no' answer on its own without any justification is worth 0 marks only.)
- 5, 4, 4, 4, 3, 2, 2, 1
 - 4, 4, 3, 3, 2, 2, 1, 1
 - 7, 4, 4, 4, 4, 4, 1, 0
- (7 marks)**