

Combinatorics II.

Discrete mathematics I – exercises

19. How many 5-digits numbers can be formed from the digits 2, 3, 4, 5 and 7, using each digit once,
a) in total; b) which are even; c) which are divisible by four?
20. How many 6-digit numbers can be formed from the digits 0, 1, 2, 3, 4 and 5, using each digit once, such that the number is divisible by 5?
21. In a factory, 500 locks are manufactured during a shift, and 4% of them are faulty. In how many different ways can we choose 10 locks such that there are
a) exactly 5 faulty locks; b) at least 2 faulty locks?
22. When rolling a die three times, how many possible results contain 6?
23. In how many different ways can we place 24 identical balls into 8 different boxes if
a) the boxes can be empty;
b) each box must contain at least 1 ball;
c) each box must contain at least 2 balls?
24. A robot is moving on the number line: in each second, it moves one unit in either direction. Starting from the origin, in how many possible ways can it reach +24 in exactly one minute?
25. There are two parallel lines, one of which has p distinct points marked, the other has q . How many triangles can be formed from the marked points as vertices?
26. A standard 52-card deck of French-suited playing cards is dealt out to four players, each getting exactly 13 cards. In how many ways can we do this
a) in total; b) if the four aces are dealt to different players;
c) if the four aces are dealt to the same player?
27. In how many ways can we distribute 30 balls into 100 boxes if each box must have either exactly 6 balls or none, and
a) all balls are identical;
b) all balls are different, and the order of balls matters inside the boxes;
c) all balls are different, and the order of balls doesn't matter inside the boxes?
28. In how many orders can we arrange the letters of MISSISSIPPI such that the four S's are not consecutive?
29. In how many orders can the numbers $1, 2, \dots, n$ be arranged such that 1 and 2 are not adjacent?

30. In how many orders can we arrange n zeros and k ones such that no two ones are adjacent?
31. How many natural numbers exist whose digits are strictly increasing?
32. In the expansion of $(a + b)^{22}$, what is the coefficient of
a) a^{22} ; b) $a^{21}b$; c) $a^{17}b^5$; d) $a^{14}b^8$?
33. In the expansion of $\left(\frac{1}{a} + a^2\right)^9$, what is the term that does not contain a ?
34. a) In the expansion of $(x^7 + 2x^3)^{27}$, what is the coefficient of x^{97} ?
b) In the expansion of $(x^{11} + 5x^4)^{57}$, what is the coefficient of x^{417} ?
c) In the expansion of $(6x^8 - 11x^5)^{32}$, what is the coefficient of x^{178} ?
35. How many zeros does $11^{100} - 1$ end in?
36. In a survey, 100 participants were asked about the types of media sources they follow. The number of people mentioning each type of media was as follows:
• internet: 65; • television: 38; • newspapers: 39;
• internet and television: 20; • internet and newspapers: 20;
• television and newspapers: 9; • internet, television and newspapers: 6.
How many of these 100 people follow neither of these sources?
37. Is it true that among 8 people there are always at least two who were born on the same day of the week?
38. What is the smallest number of people so that there are always at least four who was born in the same month?
39. At a meeting of 34 people, each participant knows at most 10 of the participants. Is it true that there must be 4 people at the meeting who know the same number of people in the meeting?
40. What is the maximum number of natural numbers such that no two of them has a difference divisible by eight?
41. A bag contains 10 red, 20 yellow and 40 green balls. How many balls do we have to choose so that they certainly contain:
a) a yellow one; b) three balls with different colours; c) three balls of the same colour;
d) five balls of the same colour;
e) 15 balls of the same colour; f) two consecutive greens?
42. In how many ways can we split 1000000 into the product of three natural numbers if the order of the factors a) matters, b) doesn't matter?
43. Solve the equation $0.7 \cdot \binom{25}{x} = \binom{23}{x}$ on the set $\{0, 1, 2, \dots, 23\}$.
44. Consider all strings of length six composed of the digits 0-9 (any number of times, including zero times). How many of these strings are such that they do not contain the substring 42 (i.e. the digits 4 and 2 in this order, next to each other)?