Combinatorics I.
Discrete mathematics I – exercises

1. In how many ways can we:
   a) arrange six different balls;
   b) arrange three yellow, two green and one red balls;
   c) choose four distinct balls from six different balls if the order matters;
   d) choose four distinct balls from six different balls if the order doesn't matter;
   e) choose four from six different balls if repetition is permitted and the order matters;
   f) choose four from six different balls if repetition is permitted and the order doesn't matter?

2. How many eight-digit numbers can be formed from the digits 1, 2, 3, 4 and 5?

3. There are 15 participants in a running race. Everyone arrives at the finish, and there is no tie.
   a) How many different outcomes are possible?
   b) How many different outcomes are possible for the first three places?

4. In how many ways can we fill a lottery ticket? (It contains 90 numbers, and we have to select 5 of them.)

5. A post office sells 12 types of cards. In how many different ways can we purchase 5 cards (assuming that the post office has enough cards of each type)?

6. In how many possible ways can 12 people be placed into three rooms, if the rooms have three, four and five beds respectively, so that everyone gets a bed?

7. How many ten-digit numbers are there with no repeated digits?

8. In how many ways can 6 people sit around a round table if we do not distinguish between orders that can be rotated into each other?

9. How many different five-digit numbers can be formed from exactly the following digits:
   a) 1, 2, 3, 4, 5;   b) 1, 1, 2, 3, 4;   c) 1, 1, 2, 2, 2?

10. How many different outcomes can we get by
    a) flipping a coin 10 times;   b) rolling a die 10 times?

11. A quiz contains 30 questions, each question has 5 possible answers, and exactly one of them must be selected. In how many possible ways can we fill the quiz?

12. In how many ways can we distribute 6 identical books among 20 students if no student gets more than one book?

13. In how many different orders can four couples sit along a bench if the couples sit next to each other?

14. How many $S \subseteq \{1, 2, 3, \ldots, 20\}$ exists
   a) ;   b) for which $1 \in S$;   c) $1 \in S \wedge 2 \in S$;   d) $1 \in S \vee 2 \in S$;
   e) $|S| = 4$;   f) $|S| = 8 \wedge 1 \in S$?

15. A bag contains six balls, labelled 1, 2, 3, 4, 5 and 6. In how many possible ways can we choose four different balls in a row:
   a) ;   b) without choosing 6;   c) if the first ball is 1;
   d) if the first ball is even;   e) if the last ball is even?

16. How many six-digit numbers are there:
   a) whose all digits are different, and none of them is 0;   b) whose all digits are different;
   c) whose adjacent digits are different;   d) which contain the digit 0;
   e) which contain the digit 0 exactly once?

17. The number of permutations of $n + 2$ distinct elements is 20 times more than the number of permutations of n distinct elements. What is n?

18. a) Write a program that takes a finite set A ($|A| \leq 6$) and prints all permutations of A.
   b) Write a program that takes a finite set A ($|A| \leq 6$) and an integer $1 \leq k \leq |A|$, and it prints all k-combinations without repetition of A.
   c) Write a program that takes a finite set A ($|A| \leq 5$) and an integer $1 \leq k \leq 4$, and it prints all k-variations with repetition of A.