Computer Science BSc
Basic Mathematics TEST-3
13-rd of December 2021

Reasoning and justification are needed in the solutions

1. *(8 points)* Using the Gauss-Jordan method (=EBT-method) determine the inverse of the following matrix:

$$A = \begin{bmatrix} 1 & 3 & 2 \\ 1 & 8 & 0 \\ 2 & 3 & 5 \end{bmatrix} \in \mathbb{R}^{3\times 3}$$

2. *(14 points)* Determine the eigenvalues and eigenvectors of the following matrix. Determine the algebraic and geometric multiplicities of the eigenvalues. Discuss the digonalizability of A (determine C and $C^{-1}AC$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & -1 \\ 0 & 1 & 2 \end{bmatrix} \in \mathbb{R}^{3\times 3}$$

3. *(12 points)* Consider the following subspace W and the vector x in $\mathbb{R}^4$:

$$W := \text{Span}\,(\ (1,1,-1,0);\ (1,1,1,-1);\ (2,1,2,1)\ )\,, \quad x := (-1,1,-2,1)$$

   a) Determine an orthogonal and an orthonormal basis in W.

   b) Decompose the vector x by the subspace W into parallel and orthogonal components.

4. *(8 points)* Consider the following $\mathbb{R} \to \mathbb{R}$ type function f:

$$f(x) := x^2 - 12x + 11 \qquad (x \in (-\infty; 5])$$

   Prove that f is invertible, and determine the sets $D_{f^{-1}}$, $R_{f^{-1}}$ and the for $y \in D_{f^{-1}}$ the function value $f^{-1}(y)$.

   (ATTENTION: "graphical" solution cannot be acceptable here.)

5. *(8 points)* Prove by definition that

$$\lim_{x\to+\infty} \frac{3x^3 - x^2 - 5x + 1}{2x^3 + x + 4} = \frac{3}{2}$$