Name: ....., *NEPTUN-code* .....

Group, teacher: .....

Points: .....

Computer Science BSc Basic Mathematics TEST-2
18-th of November, 2022

Reasoning and justification are needed in the solutions.

1. a) *(7 points)* Consider the complex numbers $z_1 = 8 - i, \quad z_2 = 3 - 2i$. Compute the value of the following expression (we ask the result in algebraic form):

$$\left(\frac{z_1}{z_2}\right)^2 \cdot (\overline{z_1 - z_2})$$

   b) *(4 points)* Solve the equation $z^3 - 3z^2 + 4z - 2 = 0$ on the set of complex numbers.

2. Let $A = \begin{bmatrix} 2 & -1 & 3 \\ 2 & 0 & 2 \end{bmatrix} \in \mathbb{R}^{2\times 3}, \quad B = \begin{bmatrix} 3 & 1 & -2 \\ 2 & 4 & 2 \end{bmatrix} \in \mathbb{R}^{2\times 3}.$

   (a) *(6 points)* $(AB^T)^{-1} \cdot (A - B) =?$
   (b) *(4 points)* $\det(A^T B) =?$

3. *(7 points)* Using cofactors (signed subdeterminants) compute the inverse of the following matrix, and check your result by matrix multiplication. *(Attention: the Gauss-Jordan elimination method is not acceptable here!)*

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 1 & 1 & 3 \end{bmatrix}$$

4. *(4 points)* Determine whether the following subset is a subspace in $\mathbb{R}^4$.

$$H := \left\{ (x,\, y,\, z,\, u) \in \mathbb{R}^4 \;\middle|\; xyzu \geq 0 \right\}$$

5. Consider the following subspace in $\mathbb{R}^4$:

$$W := \left\{ (y + z + 2u,\ x,\ x - y - u,\ x + y - 2z + u) \in \mathbb{R}^4 \,\middle|\, x, y, z, u \in \mathbb{R},\ 2x + y = z + u \right\}$$

   (a) *(8 points)* Determine a basis in W. Determine the dimension of W.
   (b) *(2 points)* Determine whether the set $G = \mathrm{Span}((0,0,0,0),\ (1,0,-1,3))$ is a subspace of the vector space W or not.

6. *(8 points)* Solve the following system of linear equations using the Gauss-Jordan method. Write the solution in scalar form and in vector form. Determine the rank of the coefficient matrix.

$$\begin{array}{rcrcrcrcrcr} 3x_1 & - & 2x_2 & - & x_3 & + & 4x_4 & + & 2x_5 & = & -2 \\ 5x_1 & + & x_2 & - & 2x_3 & + & 7x_4 & + & 4x_5 & = & -1 \\ x_1 & + & 8x_2 & - & x_3 & + & 2x_4 & + & 2x_5 & = & 4 \end{array}$$