Name: ...., *NEPTUN-code* ....

Group, teacher: ....

Points: ....

Computer Science BSc Basic Mathematics
TEST-1
22-nd of October, 2021

Reasoning and justification are needed in the solutions.

1. *(7 points)* Find the simplest form of the following expression ($a \in \mathbb{R}$, $a \neq 1$, $a \neq -1$):

$$\left(\frac{a}{a+1} + \frac{a^2+a+1}{a^3-1} + \frac{2a}{a^2-1}\right) \cdot \left(\frac{a}{a+1} + \frac{a^2+a+1}{a^3-1} - \frac{2a}{a^2-1}\right)$$

2. *(8 points)* Solve the following inequality on the set of real numbers:

$$\sqrt{x-x^2} > 2x-1$$

3. *(4+8= 12 points)*

a) Determine the real parameter a such that the number 1 will be the root of the following polynomial. Then factor out the root factor according to 1 from it.

$$P(x) := x^3 + ax^2 + x - 6 \quad (x \in \mathbb{R})$$

b) Solve the following equation on the set of real numbers:

$$\log_3(x^3 + 4x^2 + x - 6) - 2 \cdot \log_9(x+3) = 2 \cdot \log_3(\sqrt{3-x})$$

4. *(7 points)* Solve the following inequality on the set of real numbers:

$$(1-\cos x)^2 - 3\sin^2 x \leq 0$$

5. *(1+7+1= 9 points)*

a) Write down the mathematical form (using quantifiers) of the following statement :

For all great enough positive natural numbers n we have:

$$\frac{3n^5 - 2n^4 + n^3 + n^2 - n + 7}{5n^6 + 2n^5 - n^4 - n^3 + n - 398} < \frac{1}{5}$$

b) Prove – by determining a good threshold – that the above statement is true.

c) Give its negation as well.

6. *(7 points)* Using mathematical induction prove the following statement :

$$\forall\, n \in \mathbb{N}^+ : \qquad \sum_{k=1}^{n} \frac{k+2}{k \cdot (k+1) \cdot 2^k} = 1 - \frac{1}{(n+1) \cdot 2^n}$$