Functions

1.

In each of the following examples decide if the relation f is a function. If f is a function then determine the domain and range of f and decide whether f is surjective, injective and/or bijective.

(a) $A = \{1, 2, 3, 4, 5\}, B = \{10, 11, 12, 13, 14\}, f \subseteq A \times B, f = \{(1, 11), (2, 11), (4, 12), (5, 10)\}$

(b) $A = \{1, 2, 3, 4\}, B = \{a, b, c, d, e, f\}, f \subseteq A \times B, f = \{(1, a), (2, c), (3, e), (3, f), (4, a)\}$

(c) $A = \{1, 2, 3, 4, 5\}, B = \{a, b, c, d, e, f\}, f \subseteq A \times B, f = \{(1, a), (4, e), (5, d)\}$

(d) $A = \{1, 2, 3\}, B = \{1, 3, 5\}, f \subseteq A \times B, f = \{(1, 1), (2, 5), (3, 5)\}$

2.

Let $m \in \mathbb{R}^+$ and $A = \{\text{all isosceles triangles with height of } m \text{ (from base)}\}$, $B = \mathbb{R}^+$. Define the binary relation $R \subseteq A \times B$ as follows: $aRb, a \in A, b \in B$, if the area of a equals b. Show that R is a function, and examine the properties of f (i.e. decide if f is surjective, injective and/or bijective).

3.

(a) Let $f : \mathbb{R} \to \mathbb{R}, f(x) := 3x - 4$. Prove that the function f is bijective, and find the inverse of f.

(b) Let $g : \mathbb{R} \to \mathbb{R}, g(x) := 3 - |x|$. Prove that the function g is neither injective, nor surjective.

4.

In each of the following examples decide whether f is a function.

(a) $f \subseteq \mathbb{N} \times \mathbb{N}, xfy \iff x \mid y$

(b) P is the set of prime numbers, $f \subseteq P \times P, xfy \iff x \mid y$

(c) $f \subseteq \{0, 3, 5\} \times \{1, 2, 5\}, xfy \iff xy = 0$

(d) $f \subseteq \{1, 2, 5\} \times \{0, 3, 5\}, xfy \iff xy = 0$

(e) $f \subseteq \mathbb{N} \times \mathbb{N}, xfy \iff x$ has the same set of decimal digits as y

(f) $f \subseteq \mathbb{N} \times \mathbb{N}, xfy \iff 2x = y$

(g) $f \subseteq \mathbb{Z} \times \mathbb{Z}, xfy \iff x^2 = y^2$

(h) $f \subseteq \mathbb{N} \times \mathbb{N}, xfy \iff x^2 = y^2$

(i) $f \subseteq \mathbb{R} \times \mathbb{R}, xfy \iff x^2 + y^2 = 9$

5.

In each of the following examples decide if the given binary relation is a function.

(a) $f_1 = \{(x, y) \in \mathbb{R} \times \mathbb{R} \mid 7x = y^2\} \subseteq \mathbb{R} \times \mathbb{R}$

(b) $f_2 = \{(x, y) \in \mathbb{R} \times \mathbb{R} \mid x = y^2 + 6y\} \subseteq \mathbb{R} \times \mathbb{R}$

(c) $f_3 = \{(x, y) \in \mathbb{R} \times \mathbb{R} \mid 7x^2 - 6 = y\} \subseteq \mathbb{R} \times \mathbb{R}$

(d) $f_4 = \{(x, y) \in \mathbb{R} \times \mathbb{R}_0^+ \mid y = |x|\} \subseteq \mathbb{R} \times \mathbb{R}_0^+$

(e) $f_5 = \{(x, y) \in \mathbb{R} \times \mathbb{R} \mid y = (x + 4)^2\} \subseteq \mathbb{R} \times \mathbb{R}$

(f) $f_6 = \{(x, y) \in \mathbb{R} \times \mathbb{R}_0^+ \mid 2y = \sqrt{x}\} \subseteq \mathbb{R} \times \mathbb{R}_0^+$

(g) $f_7 = \{(x, y) \in \mathbb{Z} \times \mathbb{Z} \mid 7 \mid x - y\} \subseteq \mathbb{Z} \times \mathbb{Z}$

(h) $f_8 = \{(x, y) \in (\mathbb{R} \setminus \{0\}) \times (\mathbb{R} \setminus \{0\}) \mid xy = 1\} \subseteq (\mathbb{R} \setminus \{0\}) \times (\mathbb{R} \setminus \{0\})$

(i) $f_9 = \{(x, y) \in \mathbb{R} \times \mathbb{R} \mid xy = 1\} \subseteq \mathbb{R} \times \mathbb{R}$

(j) $f_{10} = \{(x, y) \in \mathbb{Z} \times \mathbb{Z} \mid |x - y| \leq 3\} \subseteq \mathbb{Z} \times \mathbb{Z}$

(k) $f_{11} = \{(x, y) \in \mathbb{R} \times \mathbb{R} \mid y(1 - x^2) = x - 1\} \subseteq \mathbb{R} \times \mathbb{R}$
(l) $f_{12} = \{(x, y) \in (\mathbb{R} \setminus \{1, -1\}) \times (\mathbb{R} \setminus \{1, -1\}) \mid y(1 - x^2) = x - 1\} \subseteq (\mathbb{R} \setminus \{1, -1\}) \times (\mathbb{R} \setminus \{1, -1\})$

For each relation that is a function decide if it is injective, surjective and/or bijective. For each relation that is not a function and is a homogeneous relation, decide if it is reflexive, symmetric and/or transitive.