Combinatorics

Discrete mathematics 1. exercises

Practice teacher: Uray M. János

1. How many possible ways are there to:

   a) arrange six different balls;
   b) arrange three yellow, two green and one red ball;
   c) pick four balls from six different one without replacement, if the order matters;
   d) pick four balls from six different one without replacement, if the order does not matter;
   e) pick four balls from six different one with replacement, if the order matters;
   f) pick four balls from six different one with replacement, if the order does not matter?

2. How many eight-digit numbers exist which contain only 1, 2, 3, 4 and 5 as digits?

3. There are 15 runners in a race. Everyone reaches the finish, and there is no tie.

   a) In how many possible orders can they arrive at finish?
   b) How many possibilities are there for the first 3 arrivals?

4. In the lottery, players choose five different numbers between 1 and 90. How many possible ways are there to play once in this lottery?

5. There are 10 kind of wares in a bakery. How many possible ways are there to buy 12 wares?

6. How many possible ways can 12 people be placed into three rooms, whose capacities are 3, 4 and 5, respectively?

7. How many ten-digit numbers exist with no repeated digits?

8. How many possible ways can 6 people sit at a circular table, if rotation does not matter?

9. How many five-digit numbers can be constructed from exactly the following digits?

   a) 1, 2, 3, 4, 5;   b) 1, 1, 2, 3, 4;   c) 1, 1, 2, 2, 2;

10. We repeat ten times
    a) flipping a coin;   b) throwing a dice.
    How many possible result sequence can we get?

11. A robot moves on the number line, moving by 1 in each step, either left or right. Starting from the origin, how many possible ways can it arrive at +24 after one minute?

12. A bag contains six balls labelled with 1, 2, 3, 4, 5 and 6, respectively. How many possible ways can four balls be picked without replacement:

a) ;   b) if 6 is never picked;   c) if 1 is picked first;

d) if an even number is picked first;   e) if an even number is picked last?

13. How many six-digit numbers are there:

a) whose all digits are different, and none is 0;   b) whose all digits are different;

c) whose adjacent digits are different;   d) which has digit 0;

e) which has exactly one digit 0?

14. How many n-ary m-value Boolean functions exist, i.e. functions of type $f : \{0, 1\}^n \to \{0, 1\}^m$?

15. How many possible ways can ten uniform balls be put into three labelled bags?

16. In the lottery, players choose five different numbers between 1 and 90. Comparing to the results of the lottery, how many possible ways are there to have:

a) 0 match;   b) 1 match;   c) 2 matches;   d) 3 matches;   e) 4 matches;   f) 5 matches?

17. How many subsets does the set $\{1, 2, 3, \ldots, 20\}$ have:

a) ;   b) which contains 1;   c) which contains both 1 and 2;

d) which contains any of 1 or 2 (or both)?

18. How many possible ways can the numbers $1, 2, \ldots, n$ be arranged such that 1 and 2 are not adjacent?

19. How many possible ways can the letters of MISSISSIPPI be arranged such that the four S are not in one continuous block?

20. In the expansion of $(a + b)^{22}$, what is the coefficient of

a) $a^{14}b^8$;   b) $a^{17}b^5$?

21. Starting from the bottom left corner of a grid of 3×10 squares, how many possible ways can the top right corner be reached if only up, right and up-right movement is allowed, moving one square in each step?

22. On two parallel straight lines, p and q distinct points are marked, respectively. How many triangles can be created using the points as vertices?

23. How many zeros does $11^{100} - 1$ end with in decimal form?

24. If order matters, how many possible ways are there to split:

a) 100 into the sum of 7 positive integers;   b) 200 into the sum of 12 natural numbers;

c) 12 into sum of ones and twos?

25. How many possible ways can 12 dominos of size 2×1 fill a grid of 2×12 squares?

26. The 52 cards of French playing cards are divided among four players, 13 for each. How many possible ways are there to do this:

a) ; b) if the four aces are divided evenly; c) if all four aces are given to one player?

27. How many possible ways can n zeros and k ones be arranged such that no two ones are adjacent?

28. A bag contains 10 red, 20 yellow and 40 green balls. How many balls must be picked to be sure that they contain:

a) yellow; b) three different colours; c) three of the same colour;
d) five of the same colour; e) 15 of the same colour; f) two adjacent greens?

29. What is the minimal number of people such that they surely include:

a) four people born in the same month;
b) at least three people born in each month?

30. Among 30 students,
   • 12 like maths; • 14 like physics; • 13 like chemistry;
   • 5 like both maths and physics; • 4 like both maths and chemistry;
   • 7 like both physics and chemistry; • and 3 like all three subjects.
   How many students like neither of these subjects?

31. a) How many natural numbers less than 100 are divisible by neither 2, nor 3, nor 5?
    b) How many natural numbers less than 1000 are divisible by neither 2, nor 3, nor 5, nor 7?

32. How many possible ways can 30 balls be arranged into 100 boxes such that each box gets either no balls or exactly 6, and:

a) the balls are the same;
b) the balls are different, but the in-box order does not matter;
c) the balls are different, and the in-box order matters?

33. How many natural numbers can be given at maximum such that the difference of neither two is divisible by 8?